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Mini Review

Chakrabarty's Tables of Random Numbers: A Comparison of Degree of Randomness with that of Some Existing Random Numbers Tables -

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ABSTRACT

A comparative study has been carried out on the degree of randomness of the two tables of random numbers due to Chakrabarty with the four tables of random numbers due to (1) Tippet, (2) Fisher & Yates, (3) Kendall & Smith and (4) Rand Corporation. The degree of randomness has been examined on the basis of the significance of difference between the observed frequencies and the corresponding expected frequencies of the 10 digits. Chi-square (χ^2) test has been applied in examining the said significance. This paper describes the examination of randomness of the six random numbers tables and a comparison of the degree of randomness of them.

Keywords: Random numbers table; Tippet; Fisher & Yates; Kendall & Smith; Rand corporation; Chakrabarty; Degree of randomness; Chi-square test

INTRODUCTION

The scientific method of selecting a random sample, that has been found to be the vital or basic work in almost every branch of experimental sciences, consists of the use of random numbers table. Several tables of random numbers have already been constructed by the renowned scientists. Some of them are due to Fisher & Yates [1], Hald [2], Hill & Hill [3], Kendall & Smith [4], Mahalanobis [5], Manfred [6], Moses & Oakford [7], Quenouille [8], RAND Corporation [9], Rao, Mitra & Matthai [10], Rohlf & Sokal [11], Royo & Ferrer [12], Snedecor and Cochran [13], Tippett [14], etc. Recently, Chakrabarty [15,16] has constructed two tables of random numbers, one for random two-digit numbers [15] and the other for random three-digit numbers [16].

Fisher & Yates had selected the numbers from the 10th to 19th digits of A.S. Thompson's 20-figure logarithmic tables and recognized them as random numbers. In choosing from those digits, an element of randomness was introduced by using playing cards for the selection of half pages of the tables and of a column between 10th to 19th and finally for allotting these digits to the 50th place in a block. The method applied in selecting the numbers, by Fisher & Yates, creates a doubt on the randomness of the numbers generated. This leads to the necessity of examining the degree of randomness of the random numbers table constructed by Fisher and Yates. Similarly, there is also necessity of examining the degree of randomness of the tables of random numbers due to Tippet, Kendall & Smith and Rand Corporation respectively. In the meantime, some studies have been made on examining the degree of randomness of the four tables of random numbers due to Tippet, Fisher & Yates, Kendall & Smith and Rand Corporation respectively [17-18]. In the current attempt, a comparative study has been carried out on the degree of randomness of the two tables of random numbers due to Chakrabarty with the four tables of random numbers due to [19] Tippet, [20] Fisher & Yates, [15] Kendall & Smith and [16] Rand Corporation. The degree of randomness has been examined on the basis of the significance of difference between the observed frequencies and the corresponding expected frequencies of the 10 digits. Chi-square (χ^2) test has been applied in examining the said significance. This paper describes the examination of randomness of the six random numbers tables and a comparison of the degree of randomness of them.

THE TEST STATISTIC USED

The Chi-square statistic test innovated by Pearson [1,2,8,9,12,21-27] which is a test statistic used for testing of the goodness of fit, has suitably been applied here in testing the degree of randomness of table of random numbers. A test of goodness of fit establishes whether or not an observed frequency distribution differs from a theoretical distribution.

The procedure of the test includes the following steps

1. Compute the value of the chi-squared (χ^2) test statistic which resembles a normalized sum of squares of the deviations between the observed frequencies and the corresponding theoretical frequencies.
2. Determine the degrees of freedom of that statistic, which is essentially the number of categories reduced by the number of parameters of the fitted distribution.
3. Choose the level of significance of the test.
4. Compare the observed (computed) value of χ^2 to the corresponding theoretical value from the chi-squared distribution with degrees of freedom and the selected level of significance.
5. Reject the null hypothesis that the difference between the observed frequency and the theoretical frequency is insignificant based on whether observed (computed) value of χ^2 exceeds the corresponding theoretical value of χ^2 .

The chi-squared test is based on the following assumptions

Assumptions-1 (Randomness of data): The sampled data are assumed to be drawn by a random sampling from a fixed distribution or population.

Assumptions-2 (Adequate sample size): The size of the sample is assumed to be sufficiently large.

Assumptions-3 (Adequate cell frequency): Frequency corresponding to each cell requires 5 or more. When this assumption is not met, Yates's correction [27,14] is applied.

Assumptions-4 (Independence): The observations are assumed to be independent of each other.

Chi-square statistic for the current study

The random numbers tables under study are as follows:

- (1) Tippet's Random Numbers Table [14]: This table consists of 10400 four-digit numbers giving in all 41600 single digits.
- (2) Fisher & Yates Random Numbers Table [1]: This table comprises 7500 random two-digit numbers giving in all 15000 single digits.
- (3) Kendall and Smith's Random Numbers Table [4]: This table consists of 25000 random four-digit numbers giving in all 100000 single digits.

Table-1: Observed value of χ^2 Statistic of Random Numbers Tables.

Number of Digits (1 st)	Value of χ^2 of Table of Random Numbers due to					
	Tippett	Fisher & Yates	Kendall & Smith	Rand Corporation	Chakrabarty (two-digit)	Chakrabarty (three-digit)
100	3.4	15.4	5.6	5.7	0	0
200	7.9	17.7	13.4	11.6	0	0
300	8.1923	18.386	7.265	9.52	0	0
400	11.25	13.1	9.75	6.416	0	0
500	14.8	6.28	10.16	8.52	0	0
600	15.6017	10.008	12.769	6.772	0	0
700	8.7705	6.55	11.687	6.285	0	0
800	5.8	6.75	11.226	7.202	0	0
900	10.5318	6.189	10.267	7.967	0	0
1000	6.42	5.48	7.84	10.36	0	0
1100	11.398	7.95	9.417	7.289	0	0
1200	13.1479	7.076	7.206	6.014	0	0
1300	10.9994	8.682	6.7687	6.614	0	0
1400	10.0284	7.995	5.643	9.729	0	0
1500	9.9269	7.644	7.026	7.815	0	0
1600	8.8126	7.961	5.174	6.961	0	0
1700	9.5345	8.2808	4.882	8.858	0	0
1800	9.398	8.702	4.167	8.798	0	0
1900	9.4842	7.714	3.147	7.62	0	0
2000	8.4	9.97	2.43	7.81	0	0
2100	6.9248	9.014	2.04	6.905	0	0
2200	4.5816	8.297	2.583	6.138	0	0
2300	4.3787	8.994	2.565	5.251	0	0
2400	4.0041	10.2405	4.225	4.842	0	0
2500	3.664	11.104	3.768	4.976	0	0
2600	5.3768	9.9448	3.7144	5.13	0	0
2700	4.2889	8.541	4.628	5.295	0	0
2800	4.6569	6.999	2.971	5.871	0	0
2900	5.7929	4.538	3.00	5.889	0	0
3000	5.6331	6.604	2.878	6.544	0	0
3100	6.0767	4.647	2.687	6.157	0	0
3200	6.4124	3.884	2.887	6.363	0	0
3300	6.7509	4.066	2.077	3.829	0	0
3400	6.8759	4.6707	2.695	3.954	0	0
3500	8.5087	4.9755	2.69	3.206	0	0
3600	6.6126	3.7949	3.071	2.832	0	0
3700	7.2001	3.358	2,944	0.024	0	0
3800	7.5998	3.1901	3.201	2.395	0	0
3900	7.3331	4.2056	4.498	2.303	0	0
4000	8.335	4.9175	4.4715	1.738	0	0
4100	8.4865	4.4239	4.312	1.668	0	0
4200	8.2709	3.8658	4.262	2.484	0	0
4300	7.7256	4.2776	5.1503	1.962	0	0
4400	10.6277	4.0041	5.2983	1.394	0	0
4500	9.53	3.7807	5.858	1.848	0	0
4600	8.027	7.2959	5.321	2.288	0	0
4700	9.932	2.4961	4.077	2.267	0	0
4800	10.103	3.245	4.013	3.216	0	0
4900	11.369	2.898	5.245	2.726	0	0
5000	11.392	2.7952	5.484	3.136	0	0
5100	10.5675	3.589	5.783	2.709	0	0
5200	11.1768	5.777	6.501	2.776	0	0
5300	11.4629	3.6998	5.412	3.558	0	0
5400	10.5303	3.8445	5.036	3.457	0	0
5500	10.4275	3.6058	5.388	3.874	0	0
5600	10.5318	5.0036	5.462	3.933	0	0
5700	11.3367	5.7851	4.681	3.841	0	0
5800	15.814	6.3243	4.092	4.438	0	0
5900	10.135	7.0341	4.307	4.608	0	0
6000	10.805	6.688	5.362	4.843	0	0

6100	9.8492	7.3731	6.423	5.587	0	0
6200	9.5807	7.4613	6.395	7.04	0	0
6300	9.4377	7.3573	5.806	6.821	0	0
6400	10.669	7.279	6.675	5.381	0	0
6500	9.3656	6.636	6.774	6.681	0	0
6600	10.458	6.2479	6.649	5.907	0	0
6700	10.372	5.7576	7.77	5.647	0	0
6800	8.3655	3.5674	8.027	5.77	0	0
6900	8.6899	5.406	6.892	6.889	0	0
7000	8.0287	3.4273	6.206	7.066	0	0
7100	7.8962	5.084	7.088	5.961	0	0
7200	8.6185	5.203	6.621	5.696	0	0
7300	8.9613	3.047	6.239	4.827	0	0
7400	8.9986	5.1216	6.565	5.289	0	0
7500	8.333	6.071	6.193	4.659	0	0
7600	9.1746	5.544	6.524	5.51	0	0
7700	9.699	4.8017	7.528	5.732	0	0
7800	8.7332	6.1348	7.336	5.661	0	0
7900	9.2667	5.2715	7.863	6.885	0	0
8000	8.7695	5.504	7.771	6.774	0	0
8100	7.3679	4.8868	6.998	6.72	0	0
8200	8.251	4.6364	6.919	7.784	0	0
8300	7.6912	4.1867	6.461	7.071	0	0
8400	7.567	3.5018	6.956	6.095	0	0
8500	6.9876	3.072	8.74	6.519	0	0
8600	7.2584	3.2025	9.522	7.955	0	0
8700	7.7045	3.7743	9.523	8.123	0	0
8800	7.4427	4.5071	10.568	8.805	0	0
8900	7.896	3.9144	9.971	7.345	0	0
9000	7.8026	4.248	9.841	6.473	0	0
9100	7.4145	4.2746	10.378	6.729	0	0
9200	8.5159	4.4044	10.162	7,748	0	0
9300	8.1087	5.6684	9.965	8.078	0	0
9400	9.3894	5.813	9.531	7.944	0	0
9500	10.404	5.8473	9.404	7.761	0	0
9600	9.7165	5.9722	9.039	9.219	0	0
9700	9.9068	4.8202	9.421	9.074	0	0
9800	9.8328	5.7298	9.067	9.615	0	0
9900	8.9808	5.0662	9.419	9.217	0	0
10000	8.296	5.432	9.391	8.442	0	0
10100	9.2524	4.992	9.03	9.132	0	0
10200	8.2079	4.8175	9.682	8.45	0	0
10300	8.0387	4.4821	10.189	8.669	0	0
10400	8.1328	3.8945	9.105	9.223	0	0
10500	8.591	3.872	8.75	8.702	0	0
10600	9.4579	4.0281	8.944	8.935	0	0
10700	9.9099	4.677	8.066	8.911	0	0
10800	9.4313	4.594	7.659	9.49	0	0
10900	8.2084	4.203	8.427	9.615	0	0
11000	6.5808	3.9865	8.571	8.815	0	0
11100	6.6414	4.3925	8.451	8.933	0	0
11200	5.848	4.651	9.253	9.187	0	0
11300	5.5923	4.5079	8.567	9.346	0	0
11400	5.0608	4.0389	7.441	9.599	0	0
11500	4.484	3.8884	7.708	9.912	0	0
11600	4.869	4.2668	8.216	10.023	0	0
11700	5.0943	5.293	8.937	9.838	0	0
11800	4.536	6.614	8.396	9.102	0	0
11900	4.531	5.7344	7.337	9.321	0	0
12000	4.3896	5.671	6.824	9.573	0	0
12100	5.287	5.4714	7.427	10.79	0	0
12200	5.052	6.1468	7.308	9.612	0	0
12300	5.073	5.469	7.215	9.704	0	0
12400	5.608	7.8212	6.288	8.875	0	0



12500	6.32	8.2478	6.32	9.192	0	0
12600	6.7166	8.509	6.30	9.843	0	0
12700	7.1556	8.8626	6.485	9.517	0	0
12800	7.4749	8.3969	7.283	8.926	0	0
12900	7.1319	8.9979	8.566	8.926	0	0
13000	6.3431	9.469	7.529	8.333	0	0
13100	6.0259	8.8581	7.889	8.057	0	0
13200	5.5684	9.664	7.999	8.389	0	0
13300	5.7254	9.766	8.239	8.849	0	0
13400	6.3911	8.4756	7.153	10.001	0	0
13500	11.8595	9.497	6,262	9.787	0	0
13600	4.9459	6.5339	6.574	10.303	0	0
13700	4.7411	9.3647	6.998	10.039	0	0
13800	3.5648	8.817	6.755	9.542	0	0
13900	4.885	8.5059	6.64	9.567	0	0
14000	12.3586	8.2736	5.635	9.076	0	0
14100	4.9539	8.566	5.127	9.158	0	0
14200	4.4883	9.5832	4.769	9.466	0	0
14300	4.3462	10.449	4.736	8.419	0	0
14400	4.1505	10.526	4.918	7.866	0	0
14500	6.1549	10.192	5.397	7.696	0	0
14600	4.4307	9.941	5.412	8.59	0	0
14700	4.1109	9.987	5.507	8.01	0	0
14800	3.449	9.725	5.161	7.612	0	0
14900	3.449	8.614	5.246	8.008	0	0
15000	3.1974	26.118	5.111	8.463	0	0
15100	3.094	-----	5.608	7.927	0	0
15200	3.617	-----	5.289	7.82	0	0
15300	3.3507	-----	4.314	7.388	0	0
15400	3.3773	-----	4.554	6.506	0	0
15500	3.3915	-----	4.276	6.058	0	0
15600	2.994	-----	3.483	7.094	0	0
15700	3.3114	-----	3.261	7.338	0	0
15800	2.3523	-----	3.445	7.343	0	0
15900	3.182	-----	3.39	6.839	0	0
16000	3.2736	-----	3.732	6.88	0	0
16100	2.9751	-----	3.492	6.991	0	0
16200	2.3912	-----	3.327	6.323	0	0
16300	2.7744	-----	2.836	6.673	0	0
16400	2.5864	-----	3.04	6.565	0	0
16500	2.4243	-----	3.171	6.777	0	0
16600	2.2081	-----	3.562	7.668	0	0
16700	2.321	-----	3.433	7.3682	0	0
16800	2.5904	-----	3.137	7.319	0	0
16900	1.8203	-----	3.536	7.578	0	0
17000	1.885	-----	3.955	7.637	0	0
17100	1.5281	-----	3.93	7.773	0	0
17200	1.5152	-----	4.226	8.28	0	0
17300	1.6973	-----	4.004	7.513	0	0
17400	1.6859	-----	4.045	6.96	0	0
17500	1.537	-----	4.129	6.765	0	0
17600	1.546	-----	4.501	6.758	0	0
17700	1.6856	-----	4.771	7.219	0	0
17800	1.546	-----	4.868	7.76	0	0
17900	2.3487	-----	4.318	7.859	0	0
18000	1.8919	-----	4.837	7.851	0	0
18100	2.282	-----	5.262	8.082	0	0
18200	2.313	-----	5.083	8.06	0	0
18300	2.549	-----	4.958	8.19	0	0
18400	2.4539	-----	4.589	7.977	0	0
18500	2.3757	-----	4.312	7.446	0	0
18600	2.4022	-----	3.887	7.512	0	0
18700	2.6152	-----	3.72	6.953	0	0
18800	2.157	-----	3.812	7.09	0	0

18900	2.656	-----	4.079	6.965	0	0
19000	3.2563	-----	3.855	5.575	0	0
19100	2.377	-----	3.852	7.087	0	0
19200	2.714	-----	4.476	7.228	0	0
19300	2.468	-----	3.652	7.469	0	0
19400	2.802	-----	3.791	7.755	0	0
19500	3.138	-----	3.256	8.146	0	0
19600	2.989	-----	3.921	8.114	0	0
19700	2.864	-----	3.474	7.868	0	0
19800	3.0296	-----	3.346	8.371	0	0
19900	2.912	-----	3.454	8.712	0	0
20000	2.798	-----	3.203	8.232	0	0
20100	3.203	-----	3.061	7.638	0	0
20200	3.637	-----	3.582	8.312	0	0
20300	3.8863	-----	3.714	7.908	0	0
20400	3.65	-----	3.89	7.932	0	0
20500	3.789	-----	3.769	6.835	0	0
20600	3.817	-----	3.629	7.956	0	0
20700	3.759	-----	3.657	7.706	0	0
20800	4.119	-----	3.635	7.956	0	0
20900	4.388	-----	3.84	8.532	0	0
21000	4.692	-----	3.634	9.224	0	0
21100	3.752	-----	3.978	9.031	0	0
21200	4.1596	-----	4.074	9.387	0	0
21300	4.0916	-----	3.962	9.174	0	0
21400	3.848	-----	3.96	9.5325	0	0
21500	3.926	-----	3.824	8.824	0	0
21600	3.724	-----	3.607	8.916	0	0
21700	5.021	-----	3.65	8.03	0	0
21800	3.382	-----	3.445	8.3197	0	0
21900	3.224	-----	3.401	8.504	0	0
22000	3.3899	-----	3.577	8.416	0	0
22100	3.5105	-----	3.576	8.729	0	0
22200	3.1577	-----	4.393	9.111	0	0
22300	3.5166	-----	4.498	9.601	0	0
22400	3.5672	-----	3.791	9.256	0	0
22500	3.3201	-----	4.632	8.53	0	0
22600	3.0983	-----	3.832	8.742	0	0
22700	2.9388	-----	3.957	9.347	0	0
22800	3.153	-----	4.306	8.844	0	0
22900	2.9298	-----	5.351	8.904	0	0
23000	2.9128	-----	3.543	9.227	0	0
23100	2.574	-----	3.185	9.44	0	0
23200	2.5741	-----	3.669	9.958	0	0
23300	2.77129	-----	3.833	10.081	0	0
23400	2.3837	-----	3.649	10.334	0	0
23500	2.4918	-----	3.713	10.214	0	0
23600	2.2985	-----	3.57	10.232	0	0
23700	2.2616	-----	3.52	9.951	0	0
23800	2.4785	-----	3.734	9.931	0	0
23900	2.274	-----	4.15	10.213	0	0
24000	2.2924	-----	4.169	10.257	0	0
24100	2.326	-----	3.766	10.613	0	0
24200	2.2385	-----	3.858	10.798	0	0
24300	2.162	-----	3.968	9.958	0	0
24400	2.3401	-----	4.298	10.019	0	0
24500	2.7055	-----	4.698	10.613	0	0
24600	2.821	-----	4.556	10.778	0	0
24700	2.8491	-----	4.24	11.301	0	0
24800	3.5152	-----	3.772	11.589	0	0
24900	2.6747	-----	3.961	11.111	0	0
25000	3.3152	-----	4.32	10.604	0	0
25100	3.8342	-----	4.372	10.405	0	0
25200	3.9595	-----	4.155	11.133	0	0



25300	4.098	-----	3.943	11.399	0	0
25400	4.3545	-----	3.903	11.031	0	0
25500	4.1754	-----	3.77	11.336	0	0
25600	3.921	-----	3.475	11.474	0	0
25700	4.3292	-----	3.281	10.904	0	0
25800	4.5341	-----	3.35	10.981	0	0
25900	4.3937	-----	3.138	10.536	0	0
26000	4.4678	-----	3.16	11.166	0	0
26100	4.4131	-----	3.164	11.307	0	0
26200	4.6464	-----	3.23	9.703	0	0
26300	4.7937	-----	2.966	10.454	0	0
26400	4.7704	-----	2.833	9.120	0	0
26500	4.7163	-----	2.582	8.994	0	0
26600	4.7927	-----	3.012	9.041	0	0
26700	4.6195	-----	3.044	9.417	0	0
26800	4.5859	-----	2.878	10.228	0	0
26900	4.5969	-----	2.69	10.544	0	0
27000	4.9096	-----	2.466	10.378	0	0
27100	4.9112	-----	2.265	10.433	0	0
27200	4.4823	-----	2.534	10.653	0	0
27300	4.667	-----	2.4009	10.594	0	0
27400	4.8887	-----	2.554	9.15	0	0
27500	4.727	-----	2.558	9.365	0	0
27600	4.7702	-----	2.65	9.472	0	0
27700	4.7248	-----	2.60	9.817	0	0
27800	4.8596	-----	2.66	9.685	0	0
27900	4.7382	-----	2.92	9.44	0	0
28000	4.7695	-----	2.496	9.274	0	0
28100	4.4433	-----	2.742	9.67	0	0
28200	4.5722	-----	2.797	9.464	0	0
28300	3.9653	-----	2.737	9.512	0	0
28400	3.9282	-----	2.721	9.413	0	0
28500	4.0976	-----	2.929	9.738	0	0
28600	4.4385	-----	2.989	10.333	0	0
28700	5.487	-----	3.188	10.829	0	0
28800	4.1992	-----	3.036	10.354	0	0
28900	4.258	-----	3.071	9.647	0	0
29000	4.6386	-----	3.387	9.335	0	0
29100	4.3189	-----	3.46	9.494	0	0
29200	4.1495	-----	3.45	9.82	0	0
29300	4.6732	-----	3.88	9.217	0	0
29400	4.6489	-----	3.537	9.676	0	0
29500	5.2756	-----	3.427	9.363	0	0
29600	4.9919	-----	3.61	9.431	0	0
29700	4.9808	-----	3.612	9.841	0	0
29800	5.2645	-----	3.662	9.973	0	0
29900	5.4562	-----	3.428	10.743	0	0
30000	6.0744	-----	3.32	9.997	0	0
30100	5.9296	-----	3.396	11.192	0	0
30200	6.1662	-----	3.773	11.337	0	0
30300	5.8754	-----	3.936	11.387	0	0
30400	6.4819	-----	4.124	11.942	0	0
30500	6.1508	-----	4.082	12.226	0	0
30600	6.3709	-----	4.433	12.248	0	0
30700	6.3003	-----	4.656	12.03	0	0
30800	6.5163	-----	4.51	11.722	0	0
30900	6.7915	-----	4.206	11.464	0	0
31000	6.4007	-----	4.048	12.093	0	0
31100	6.6209	-----	4.028	12.253	0	0
31200	6.5731	-----	4.26	11.696	0	0
31300	6.6937	-----	4.209	11.715	0	0
31400	6.8152	-----	4.2	11.779	0	0
31500	6.901	-----	4.47	11.965	0	0
31600	6.7705	-----	4.224	11.854	0	0

31700	6.9166	-----	4.634	11.576	0	0
31800	6.307	-----	5.018	11.642	0	0
31900	6.7621	-----	5.05	10.801	0	0
32000	6.1931	-----	4.709	10.67	0	0
32100	6.0795	-----	4.987	10.111	0	0
32200	6.0855	-----	4.573	9.891	0	0
32300	6.0664	-----	4.93	9.596	0	0
32400	5.8082	-----	5.061	9.681	0	0
32500	6.07	-----	4.87	9.719	0	0
32600	6.257	-----	4.766	9.27	0	0
32700	6.2384	-----	4.927	9.129	0	0
32800	6.4827	-----	5.44	9.113	0	0
32900	6.0148	-----	5.315	9.608	0	0
33000	6.5653	-----	4.881	9.636	0	0
33100	6.6835	-----	5.213	9.82	0	0
33200	7.1698	-----	5.588	9.586	0	0
33300	7.7898	-----	5.998	9.906	0	0
33400	7.9719	-----	4.898	10.065	0	0
33500	7.7937	-----	5.328	9.973	0	0
33600	8.5374	-----	5.384	10.017	0	0
33700	8.9082	-----	5.127	9.799	0	0
33800	9.5029	-----	5.056	10.061	0	0
33900	9.2077	-----	5.382	10.367	0	0
34000	9.4735	-----	4.88	9.546	0	0
34100	9.0944	-----	5.014	9.628	0	0
34200	8.7398	-----	4.962	9.559	0	0
34300	8.6688	-----	5.245	9.909	0	0
34400	8.8238	-----	5.517	9.438	0	0
34500	8.7549	-----	5.094	9.444	0	0
34600	9.7251	-----	5.408	9.233	0	0
34700	9.9682	-----	5.875	10.254	0	0
34800	9.6288	-----	6.093	12.518	0	0
34900	10.2195	-----	6.172	9.902	0	0
35000	9.9896	-----	6.114	9.423	0	0
35100	10.6078	-----	6.065	9.44	0	0
35200	9.9749	-----	5.793	10.023	0	0
35300	9.8425	-----	5.662	10.224	0	0
35400	8.1661	-----	5.487	10.068	0	0
35500	10.2705	-----	6.06	10.57	0	0
35600	10.3578	-----	6.084	9.935	0	0
35700	10.2734	-----	5.928	10.063	0	0
35800	10.2379	-----	5.734	9.848	0	0
35900	10.0995	-----	5.832	10.104	0	0
36000	9.7082	-----	5.516	10.257	0	0
36100	36100	-----	5.362	10.095	0	0
36200	36200	-----	5.592	9.434	0	0
36300	36300	-----	5.763	9.136	0	0
36400	36400	-----	6.084	9.053	0	0
36500	36500	-----	5.674	8.968	0	0
36600	36600	-----	5.722	8.919	0	0
36700	36700	-----	5.505	8.945	0	0
36800	36800	-----	5.522	8.943	0	0
36900	36900	-----	5.726	8.72	0	0
37000	37000	-----	5.853	8.437	0	0
37100	37100	-----	6.11	8.035	0	0
37200	37200	-----	5.94	8.167	0	0
37300	37300	-----	6.06	7.672	0	0
37400	37400	-----	6.401	7.923	0	0
37500	37500	-----	6.129	7.614	0	0
37600	37600	-----	5.90	7.731	0	0
37700	37700	-----	6.15	7.524	0	0
37800	37800	-----	5.884	7.375	0	0
37900	37900	-----	6.371	7.827	0	0
38000	38000	-----	6.317	7.531	0	0



38100	38100	-----	6.34	7.327	0	0
38200	38200	-----	6.838	7.632	0	0
38300	38300	-----	6.757	7.545	0	0
38400	38400	-----	6.731	7.403	0	0
38500	38500	-----	6.871	7.585	0	0
38600	38600	-----	6.911	7.509	0	0
38700	38700	-----	7.025	7.57	0	0
38800	38800	-----	6.686	7.787	0	0
38900	38900	-----	6.98	7.519	0	0
39000	39000	-----	5.942	7.379	0	0
39100	39100	-----	6.994	7.446	0	0
39200	39200	-----	7.599	7.276	0	0
39300	39300	-----	7.774	7.42	0	0
39400	39400	-----	7.779	6.752	0	0
39500	39500	-----	7.616	7.282	0	0
39600	39600	-----	7.837	8.175	0	0
39700	39700	-----	7.965	6.732	0	0
39800	39800	-----	7.967	6.878	0	0
39900	39900	-----	7.868	7.026	0	0
40000	40000	-----	7.829	7.251	0	0
40100	8.6943	-----	7.914	7.315	0	0
40200	8.6632	-----	7.973	7.405	0	0
40300	8.921	-----	8.182	7.648	0	0
40400	9.0153	-----	8.318	7.997	0	0
40500	9.100	-----	8.229	8.034	0	0
40600	8.8044	-----	8.574	7.909	0	0
40700	9.3351	-----	8.469	8.276	0	0
40800	9.1598	-----	7.919	8.359	0	0
40900	9.5824	-----	7.80	8.029	0	0
41000	9.3375	-----	8.034	7.836	0	0
41100	9.5207	-----	8.636	7.529	0	0
41200	9.866	-----	8.604	7.527	0	0
41300	9.9955	-----	8.992	7.322	0	0
41400	9.9817	-----	9.303	7.695	0	0
41500	10.1614	-----	9.694	7.442	0	0
41600	10.3015	-----	10.04	7.106	0	0
41700	-----	-----	9.605	7.535	0	0
41800	-----	-----	9.391	7.656	0	0
41900	-----	-----	9.021	7.665	0	0
42000	-----	-----	8.861	7.574	0	0
42100	-----	-----	8.972	7.691	0	0
42200	-----	-----	9.014	7.817	0	0
42300	-----	-----	8.921	7.921	0	0
42400	-----	-----	8.714	7.77	0	0
42500	-----	-----	8.756	7.781	0	0
42600	-----	-----	8.724	7.802	0	0
42700	-----	-----	8.824	7.383	0	0
42800	-----	-----	8.627	7.796	0	0
42900	-----	-----	8.536	7.362	0	0
43000	-----	-----	8.514	7.262	0	0
43100	-----	-----	8.66	7.239	0	0
43200	-----	-----	8.23	7.171	0	0
43300	-----	-----	8.259	7.469	0	0
43400	-----	-----	8.465	7.48	0	0
43500	-----	-----	8.854	7.465	0	0
43600	-----	-----	8.663	7.796	0	0
43700	-----	-----	9.197	7.455	0	0
43800	-----	-----	8.634	7.309	0	0
43900	-----	-----	8.717	6.981	0	0
44000	-----	-----	8.862	6.959	0	0
44100	-----	-----	8.854	7.127	0	0
44200	-----	-----	8.695	7.052	0	0
44300	-----	-----	8.88	7.046	0	0
44400	-----	-----	8.656	7.177	0	0

44500	-----	-----	8.769	7.375	0	0
44600	-----	-----	9.059	7.228	0	0
44700	-----	-----	9.463	7.286	0	0
44800	-----	-----	9.522	7.615	0	0
44900	-----	-----	9.91	8.005	0	0
45000	-----	-----	10.16	7.693	0	0
45100	-----	-----	10.272	7.905	0	0
45200	-----	-----	10.38	7.394	0	0
45300	-----	-----	10.18	7.458	0	0
45400	-----	-----	10.378	7.441	0	0
45500	-----	-----	10.54	7.551	0	0
45600	-----	-----	10.66	7.417	0	0
45700	-----	-----	9.959	7.236	0	0
45800	-----	-----	9.52	7.72	0	0
45900	-----	-----	10.004	7.352	0	0
46000	-----	-----	9.91	7.382	0	0
46100	-----	-----	9.95	7.829	0	0
46200	-----	-----	9.161	7.649	0	0
46300	-----	-----	9.416	7.395	0	0
46400	-----	-----	9.42	7.535	0	0
46500	-----	-----	9.368	7.145	0	0
46600	-----	-----	9.598	7.274	0	0
46700	-----	-----	9.94	7.194	0	0
46800	-----	-----	9.372	7.319	0	0
46900	-----	-----	9.555	7.414	0	0
47000	-----	-----	9.754	7.149	0	0
47100	-----	-----	9.147	7.431	0	0
47200	-----	-----	9.308	7.537	0	0
47300	-----	-----	9.844	7.71	0	0
47400	-----	-----	9.473	7.826	0	0
47500	-----	-----	9.279	7.469	0	0
47600	-----	-----	10.207	7.208	0	0
47700	-----	-----	10.692	7.537	0	0
47800	-----	-----	-----	7.77	0	0
47900	-----	-----	-----	7.856	0	0
48000	-----	-----	-----	8.435	0	0
48100	-----	-----	-----	7.13	0	0
48200	-----	-----	-----	7.265	0	0
48300	-----	-----	-----	7.42	0	0
48400	-----	-----	-----	7.539	0	0
48500	-----	-----	-----	7.868	0	0
48600	-----	-----	-----	7.498	0	0
48700	-----	-----	-----	7.402	0	0
48800	-----	-----	-----	7.248	0	0
48900	-----	-----	-----	8.189	0	0
49000	-----	-----	-----	6.433	0	0
49100	-----	-----	-----	6.542	0	0
49200	-----	-----	-----	7.339	0	0
49300	-----	-----	-----	7.597	0	0
49400	-----	-----	-----	7.824	0	0
49500	-----	-----	-----	7.779	0	0
49600	-----	-----	-----	7.881	0	0
49700	-----	-----	-----	7.927	0	0
49800	-----	-----	-----	7.623	0	0
49900	-----	-----	-----	7.576	0	0
50000	-----	-----	-----	7.271	0	0

(4) Random Numbers Table by Rand Corporation [9]: This table consists of one million single

digits consisting of 200000 random numbers of five-digits each.

(5) Chakrabarty's Table of Random Two-digit Numbers [15]: This table consists of 10000 random

Table-2:

Name of the Random Numbers Table	Rank with respect to the Degree of Randomness
Due to Fisher & Yates	5
Due to Tippet	4
Due to Kendall & Smith	3
Due to Rand Corporation	2
Due to Chakrabarty (both the tables)	1

two-digit numbers giving in all 20000 single digits.

(6) Chakrabarty’s Table of Random Three-digit Numbers [16]:
This table consists of 20000

random three-digit numbers giving in all 60000 single digits.

Let us consider Fisher and Yates random number table. This table consists of a total of 15000 single digits comprising of 7500 two-digit numbers viz.

00 , 01 , 02 , , 98 , 99

The test required in this study is to test whether the occurrences of the numbers appeared in the table is random. This is equivalent to a test to looked to make sure that equal numbers of 0s, 1s, 2s, 3s, , 9s are present in the table.

Let the number of occurrences of the ten digits in the table be N .

Let

O_i = Observed frequency of the digit i

& E_i = Expected frequency of the digit i

($i = 0, 1, 2, \dots, 9$)

among the N occurrences.

Then the χ^2 statistic for testing the null hypothesis

“the occurrences of the digits in the table is random”

i.e. “each digit has the probability 0.1 to occur in any position”

which is equivalent to testing of the null hypothesis that

“the discrepancy between the observed frequencies and the corresponding expected frequencies of the digits is insignificant”

is

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

which follows χ^2 distribution with 9 degrees of freedom.

This statistic can be applied to test the randomness of the whole table or of any part of the table.

This statistic can similarly be applied in testing of the randomness of the other five tables.

It is mentioned here that the assumptions under which chi-square test is applicable hold good in the case of the occurrences of the numbers in each of the four tables.

In the case of each of the six tables of random numbers, frequency test has been applied to

1st 100, 1st 200, 1st 300, , 1st 15000,

occurrences of the ten digits in the table.

The observed values of Chi-square Statistic obtained have been shown in (Table-1).

FINDINGS AND DISCUSSIONS

The following observations/findings have been obtained:

(1) In the case of Fisher & Yates Random Numbers Table, the highest observed value of chi-square with 9 degrees of freedom is 26.118. The theoretical value of chi-square with 9 degrees of freedom at 5% & 1% level of significance are 16.919 & 21.666 respectively. Thus, the lack of randomness of Fisher & Yates Random Numbers Table can be regarded as significant not only at 5% level of significance but also at 1% level of significance. However, the observed value of chi-square corresponds to its theoretical value at 0.055% level of significance. Thus, the lack of randomness of Fisher & Yates Random Numbers Table can be regarded as insignificant up to 0.055% level of significance and significant at the level of significance $< 0.055\%$.

(2) In the case of Tippet’s Random Numbers Table, the highest observed value of chi-square with 9 degrees of freedom is 15.814. The theoretical value of chi-square with 9 degrees of freedom at 5% level of significance is 16.919. Thus, the lack of randomness of Tippet’s Random Numbers Table can be regarded as insignificant at 5% level of significance. However, the observed value of chi-square corresponds to its theoretical value at 7.5% level of significance. Thus, the lack of randomness of Tippet’s Random Numbers Table can be regarded as insignificant up to 7.5% level of significance and significant at the level of significance $< 7.5\%$.

(3) In the case of Kendall & Smith’s Random Numbers Table, the highest observed value of chi-square with 9 degrees of freedom is 13.4 which is less than the corresponding theoretical value of chi-square at 5% level of significance. Thus, the lack of randomness of Kendall & Smith’s Random Numbers Table can be regarded as insignificant at 5% level of significance. However, the observed value of chi-square with 9 degrees of freedom namely 13.4 corresponds to the theoretical value of chi-square with 9 degrees of freedom at 18.1% level of significance. Thus, the lack of randomness of Kendall & Smith’s Random Numbers Table can be regarded as insignificant up to 18.1% level of significance and significant at the level of significance $< 18.1\%$.

(4) In the case of Random Numbers Table due to Rand Corporation, the highest observed value of chi-square with 9 degrees of freedom is 12.518 which is less than the corresponding theoretical value of chi-square at 5% level of significance. Thus, the lack of randomness of Random Numbers Table due to Rand Corporation can be regarded as insignificant at 5% level of significance. However, the observed value of chi-square with 9 degrees of freedom namely 12.518 corresponds to the theoretical value of chi-square with 9 degrees of freedom at 24% level of significance. Thus, the lack of randomness of Random Numbers Table due to Rand Corporation can be regarded as insignificant up to 24% level of significance and significant at the level of significance $< 24\%$.

(5) In the case of Random Numbers Table (of two-digit numbers) due to Chakrabarty, the observed values of chi-square have been found to be 0. Thus, there is no lack of randomness in this table and thus the table can be treated as properly random.

(6) In the case of Random Numbers Table (of three-digit numbers) due to Chakrabarty, the observed values of chi-square have also been found to be 0. Thus, there is no lack of randomness in this table and thus this table also can be treated as properly random.

From the findings obtained, one can conclude that the degree of the lack of randomness is highest (in other words, the degree of randomness is lowest) in the Fisher & Yates Random Numbers Table among the six tables of random numbers examined. On the other hand, the degree of the lack of randomness is lowest (in other words, the degree of randomness is highest) in the two Random Numbers Tables due to Chakrabarty among the six tables of random numbers examined. The six tables can be ranked with respect to the degree of randomness as follows:

The findings obtained are based on frequency test using chi-square statistic. However, there exist some other methods of testing of randomness. There is necessity of obtaining more confidence on the findings by applying the other methods of testing of randomness. Thus one problem for researcher, at this stage, is to make attempt of studying the degree of randomness of these six tables of random numbers by the other testing methods.

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